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Abstract. The coupling functions for underground cosmic ray muon component variations have been calculated, applying the CKP interaction model and allowing for the effect of muon energy loss fluctuations. For that purpose, the survival probabilities of muons have been calculated for standard rock for the range of depth 20–1000 hg and sea level energies 3·8–3000 GeV using a Monte Carlo method.

The average effect of range fluctuations was found to reduce the coupling functions by a factor between 0·96 and 0·45 over the range of depths considered. At a depth of 60 hg, the influence of such fluctuations on the diurnal variation in the muon component was found to decrease the amplitude by 0·34 % and to increase the phase by 0·26 %.

1. Introduction

The measurements of diurnal variation in the cosmic ray muon component underground are relevant both to the shape of the primary variation spectrum and its cut-off energy. To determine the parameters of free-space variation from these measurements, use is made of the coupling functions for muons for various absorber thicknesses.

At high primary energies beyond the geomagnetic sensitive region, the muon coupling functions have been evaluated by Dorman (1963) using nuclear cascade calculations and by Ahluwalia and Ericksen (1971) using a simple description of the muon component diffusion through the atmosphere. Evaluation of these functions has also been made by El Bedewi and Goned (1971) by applying the CKP interaction model of Cocconi *et al* (1961) to calculate the muon yield functions and integral intensities for various absorbing thicknesses in general. In order to apply such functions for underground measurements, one has to take into consideration the fluctuations in muon energy loss in rock which have proved to have considerable effect on the muon intensity particularly at large depths (eg Menon and Ramana Murthy 1967, Osborne *et al* 1968).

The purpose of this paper is to investigate the effect of muon range fluctuations on the coupling functions and the consequent effect on the amplitude and phase of diurnal variations detected underground.

2. Theoretical treatment

The coefficient of coupling between a primary variation at energies between E and $E+\mathrm{d}E$ and that in the muon component observed under absorber thickness equivalent to muon energy ΔE_μ, may be written in the form (Dorman 1963):

$$C(E, \Delta E_\mu) = \frac{D(E)S(E, \Delta E_\mu)}{I(\Delta E_\mu)}, \qquad (1)$$

where $D(E)\,dE$ is the primary cosmic ray differential spectrum, $S(E, \Delta E_\mu)$ the muon yield function and $I(\Delta E_\mu)$, the integral muon intensity under the absorber thickness ΔE_μ, is given by

$$I(\Delta E_\mu) = \int_{R_c}^{\infty} D(E)S(E, \Delta E_\mu)\,dE \tag{2}$$

where R_c is the cut-off rigidity for the location of observation. Accordingly the integral intensities given by the above equation depend on the model to be used in calculating the yield functions. Following El Bedewi and Goned (1971), we adopt the CKP interaction model in calculating the muon yield function given by

$$S(E, \Delta E_\mu) = \int_{\Delta E_\mu}^{\infty} M(E, E_\mu)\,dE_\mu. \tag{3}$$

In this case, the differential yield function is given by

$$M(E, E_\mu)\,dE_\mu = N(E, E_\pi)g(E_\pi)W(E_\mu)\,dE_\pi \tag{4}$$

where $g(E_\pi)$ and $W(E_\mu)$ and the pion decay and muon survival probabilities in the atmosphere, respectively, and $N(E, E_\pi)\,dE_\pi$ is the pion production spectrum as expressed by the CKP relation (Brooke *et al* 1964). Accordingly, if muon energy loss fluctuations underground are ignored, then the yield function at depth x will be

$$S_{NF}(E, x) = \int_{E_x}^{\infty} M(E, E_\mu)\,dE_\mu \tag{5}$$

where E_x is the energy corresponding to depth x on the muon energy–mean range curve, for the particular type of rock traversed. On the other hand, the above function with the effect of range fluctuations included will be given by

$$S_F(E, x) = \int_{E_m}^{\infty} P(E_\mu, x)M(E, E_\mu)\,dE_\mu, \tag{6}$$

where E_m is the minimum energy for a muon to have any chance to survive to depth x and $P(E_\mu, x)$ is the probability of a muon of sea-level energy E_μ surviving to that depth. The ratio of the coupling function at depth x with energy loss fluctuations to that without them is therefore given as

$$r = \frac{C_F}{C_{NF}} = \frac{I_{NF}(x)}{I_F(x)}\frac{S_F(E, x)}{S_{NF}(E, x)} = \frac{1}{F(x)}\frac{S_F(E, x)}{S_{NF}(E, x)} \tag{7}$$

where $F(x)$ is the enhancement factor for the muon integral intensity at depth x.

3. Muon survival probabilities underground

For determining the yield functions $S_F(E, x)$ underground, it is necessary to calculate the survival probabilities $P(E_\mu, x)$ for various depths. Such probabilities have been calculated in the present work using a Monte Carlo method for depths up to 10^3 hg of standard rock. In these calculations, the main processes by which muons lose energy were considered to be ionization (and excitation), direct pair production, bremsstrahlung and photonuclear interactions. Ionization and pair production losses were considered to be continuous and were evaluated by the expressions given by Sternheimer (1956) and

Mando and Ronchi (1952), respectively. For the discontinuous losses of bremsstrahlung and photonuclear interactions, it is customary to take the cross sections in the form (Osborne *et al* 1968)

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}v}\right)_{\mathrm{b}} = \text{constant} \times \frac{1}{v}(v^2 - \tfrac{4}{3}v + \tfrac{4}{3}) \tag{8}$$

and

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}v}\right)_{\mathrm{n}} = \text{constant} \times \frac{1}{v} \ln \frac{1}{v} \tag{9}$$

where v is the fraction of the muon energy transferred in the interaction. However, for the relatively shallow depths considered in the present investigation, it is appropriate to use approximate cross sections (eg Hayman *et al* 1963), for which the probability of energy transfer v per $\mathrm{g\,cm^{-2}}$ for either process will be

$$\phi_{\mathrm{b,n}}(v)\,\mathrm{d}v = B_{\mathrm{b,n}} \frac{1}{v}\,\mathrm{d}v \tag{10}$$

where B (in $\mathrm{g^{-1}\,cm^2}$) is the average fraction of muon energy transfer.

In the present calculations, the energy fraction v lost by a muon in a discontinuous process was randomly selected using the expression

$$v = \exp\left(-\frac{s}{(B_{\mathrm{b}} + B_{\mathrm{n}})\,\Delta x}\right) \tag{11}$$

where Δx is the depth interval (in $\mathrm{g\,cm^{-2}}$) and s is a random number uniformly distributed between 0 and 1.

The adopted values of B were those given for standard rock by Hayman *et al* (1963), namely, $B_{\mathrm{b}} = 1{\cdot}7 \times 10^{-6}$ and $B_n = 0{\cdot}7 \times 10^{-6}\ \mathrm{g^{-1}\,cm^2}$. Depth intervals Δx of 5, 10 and 50 hg were used for the depth ranges 20–50, 60–200 and 200–1000 hg respectively and 1000 particles were traced for each of the 83 muon sea-level energies selected in the range 3·8–3000 GeV. The results of these calculations are given in figure 1.

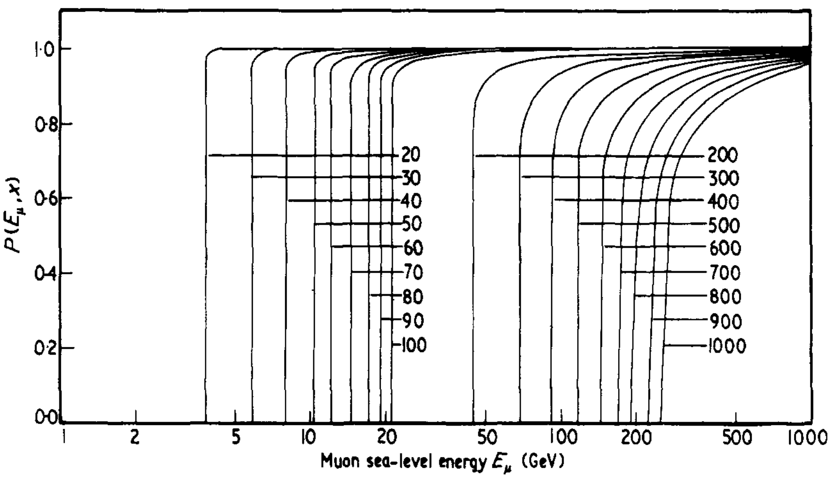

Figure 1. Survival probabilities of muons for various underground depths x (in hg).

It should be noted that in the present calculations, no allowance is made for the cut-off in photonuclear cross section at the energy transfer of 0·14 GeV (Cassiday 1971). However, investigation of the effect of such an approximation on the survival probabilities showed that it is quite small in the ranges of energies and depths considered, amounting to only 1 % at the depth of 10^3 hg.

4. The coupling functions

The calculated survival probabilities have been used to evaluate the yield functions $S_F(E, x)$ taking E_m to correspond to the energy lost due to continuous losses only. The integral muon intensities are then calculated using expression (2) after replacing ΔE_μ by the depth x and taking the primary cosmic ray differential spectrum in the form

$$D(E)\,dE = AE^{-\gamma}\,dE \qquad (12)$$

with $A = 1{\cdot}35$ and $\gamma = 2{\cdot}58$ (Brooke *et al* 1964). The enhancement factors $F(x)$ calculated from the integral intensities could be approximated as $F(x) = 1{\cdot}0$ for $x < 600$ hg and $F(x) = 1{\cdot}13$ for x between 600 and 1000 hg. The results thus obtained for the ratio r (equation (7)) and the modified coupling functions $C_F(E, x)$ are shown for various depths in figures 2 and 3 respectively.

It can be seen from figure 2 that the allowance for muon energy loss fluctuations significantly reduces the coupling functions particularly at relatively large depths underground. For example, the ratios of the mean values of coupling functions $\bar{C}_F/\bar{C}_{NF}$ taken over the relevant primary energy ranges are found to be 0·96 and 0·45 at depths of 20 and 1000 hg, respectively.

5. Effect on diurnal variation amplitude and phase

To investigate the effect of range fluctuations on the parameters of muon underground diurnal variation; the amplitudes and phases of that variation have been calculated for

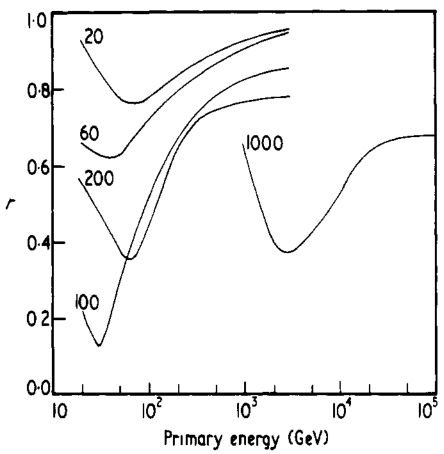

Figure 2. The ratio $r = C_F/C_{NF}$ for various depths underground (in hg).

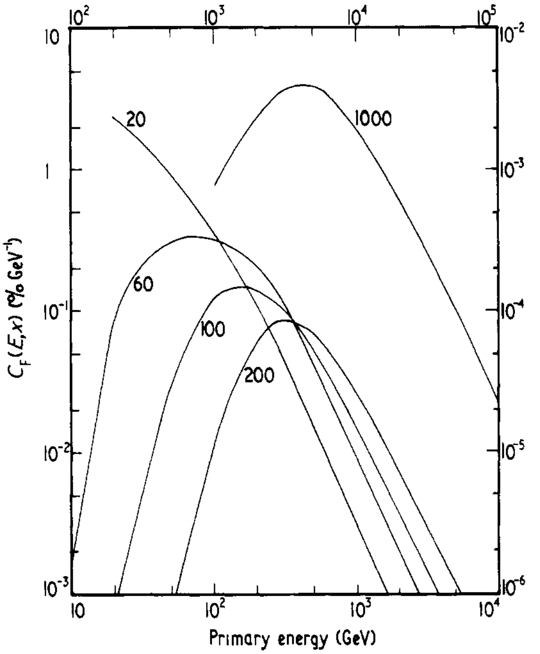

Figure 3. Coupling functions for various underground depths (in hg). Right-hand and upper scales apply for the 1000 hg depth.

various depths for low latitude (London) and medium latitude (Rome) stations. In these calculations, both coupling functions with and without range fluctuations are used applying the variational coefficients method of McCracken *et al* (1965) and assuming a rigidity independent free-space amplitude and a cut-off primary energy for the variation of 95 GeV (Ahluwalia and Ericksen 1971). As an illustration, the results obtained at a depth of 60 hg are presented in table 1, where the amplitudes A_F and A_{NF} are expressed in per cent of the free-space one, while the phases ϕ_F and ϕ_{NF} are given in local time (LT).

Table 1. Amplitudes and phases of muon diurnal variation at 60 hg underground.

Station	Amplitude (%)		Phase (LT)(h)	
	A_F	A_{NF}	ϕ_F	ϕ_{NF}
London	63·35	63·68	15·02	14·97
Rome	73·34	73·46	15·99	15·96

It can be seen from table 1 that the effect of range fluctuations at a depth of 60 hg is to reduce the amplitude of diurnal variation by 0·516 % and 0·163 % and to increase its phase by 0·33 % and 0·18 % for the first and second stations, respectively. The average effect for the two stations is the decrease of amplitude and increase of phase by about 0·34 % and 0·26 % respectively.

6. Conclusions

The fluctuations of energy loss of muons underground were found to produce a reduction in the coupling functions that becomes more significant with increasing depth. These fluctuations have the effect of decreasing the amplitude and increasing the phase of the observed diurnal variation in the muon component detected underground. The magnitude of such an effect is small at shallow depths and depends on the model used in describing the propagation and development of cosmic ray muons through the atmosphere and underground. Allowance for muon range fluctuations, however, would have a significant effect on the parameters of cosmic ray muon variations detectable at relatively large depths underground.

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